# Gears
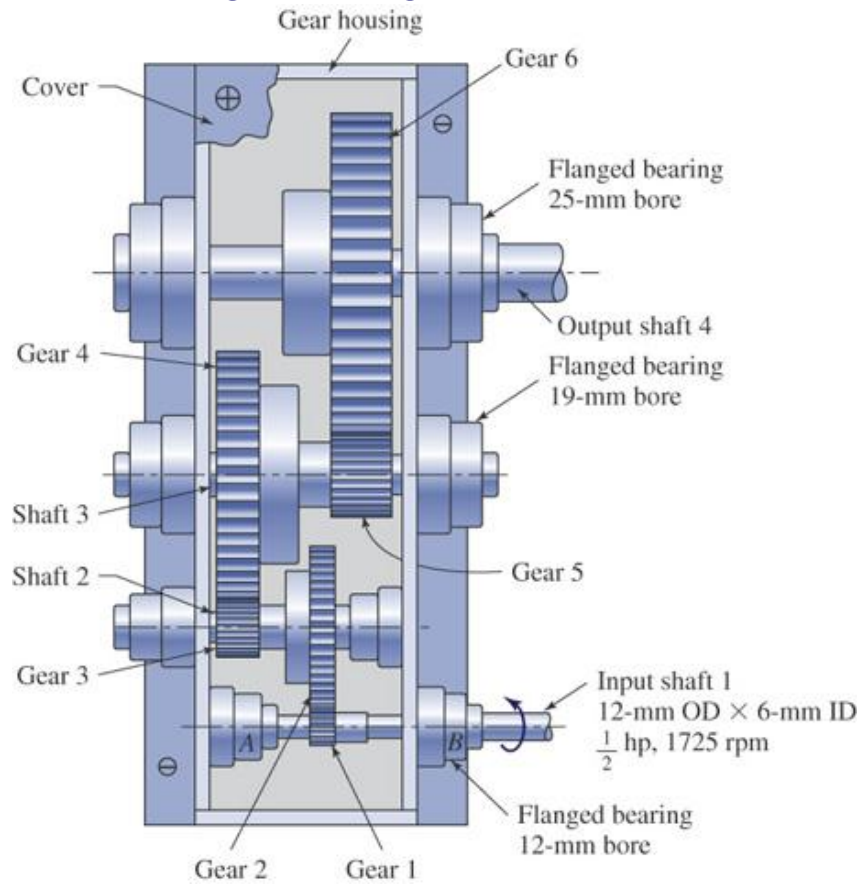
## What we need to Know about them.

1. Type of gears
2. Terminologies or nomenclatures
3. Forces transmitted
4. Design of a gear box
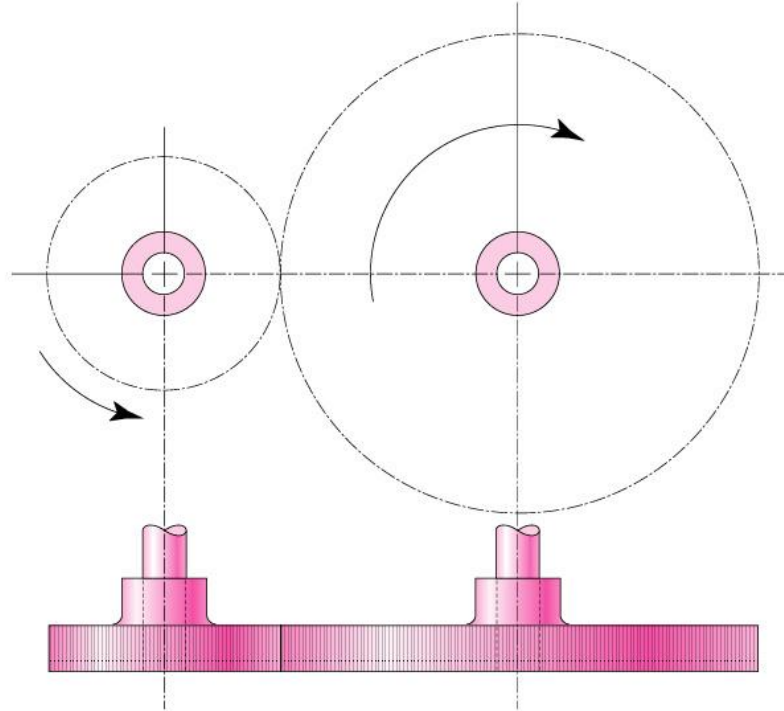


Gear housing

Cover

Gear 6

Flanged bearing
25-mm bore

Output shaft 4

Gear 4

Flanged bearing
19-mm bore

Shaft 3

Shaft 2

Gear 5

Gear 3

Input shaft 1
12-mm OD × 6-mm ID
$\frac{1}{2}$ hp, 1725 rpm

Flanged bearing
12-mm bore

Gear 2        Gear 1

# Type of Gears

- Spurs

- Helical

- Bevel

- And Worm Gears

# Spur Gears

Are used in transmitting torque between parallel shafts

# Helical Gears
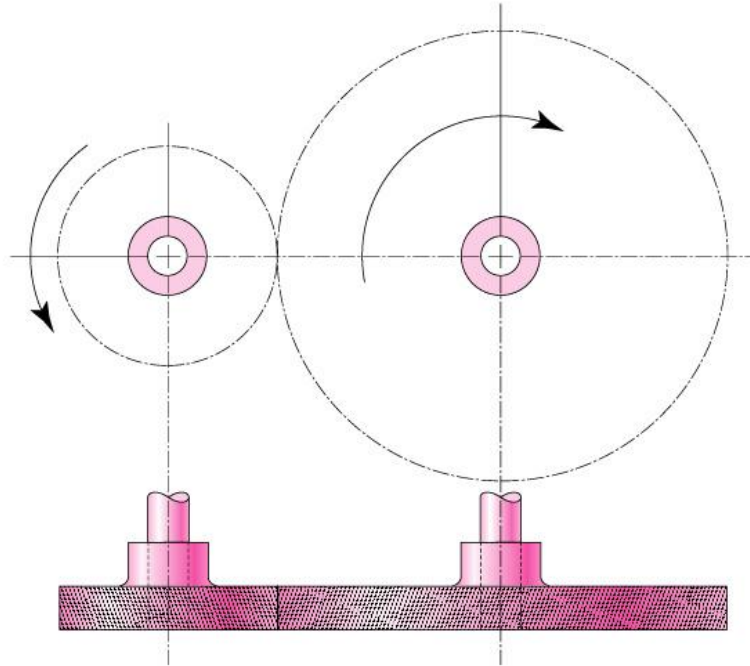
Are used in transmitting torques between parallel or non parallel shafts, they are not as noisy as spur gears
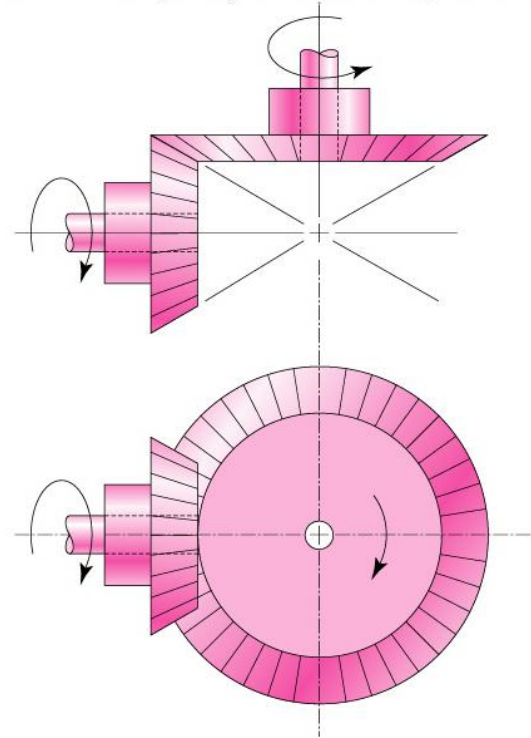
Fig. 13.2
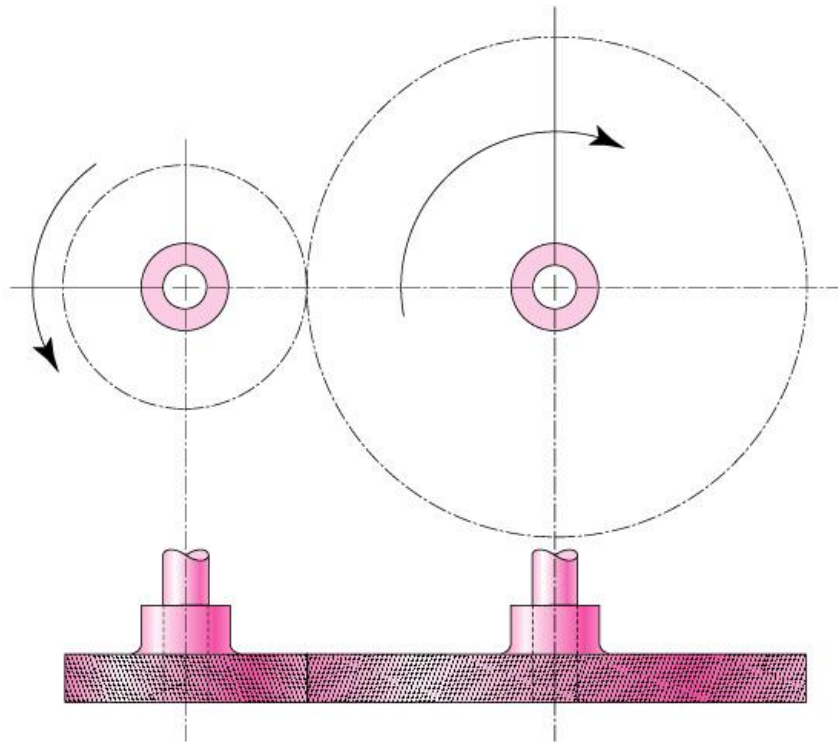
# Bevel Gears

- Are used to transmit rotary motion between intersecting shafts

Teeth are formed on conical surfaces, the teeth could be straight or spiral.
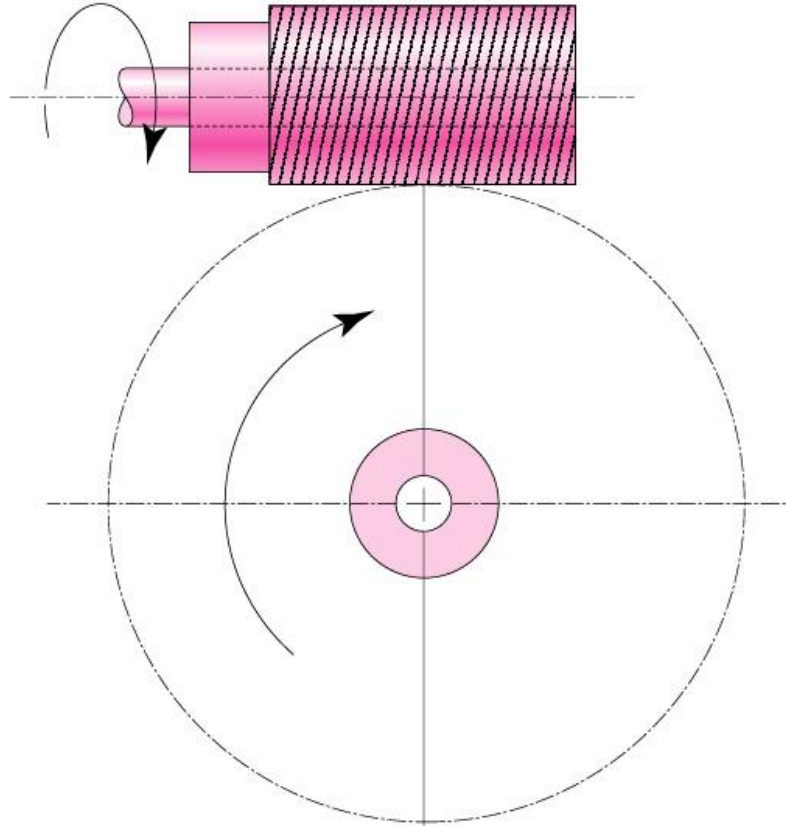
# Worm Gears

Are used for transmitting motion between non parallel and non transmitting shafts, Depending on the number of teeth engaged called single or double. Worm gear mostly used when speed ratio is quiet high, 3 or more

# Nomenclature
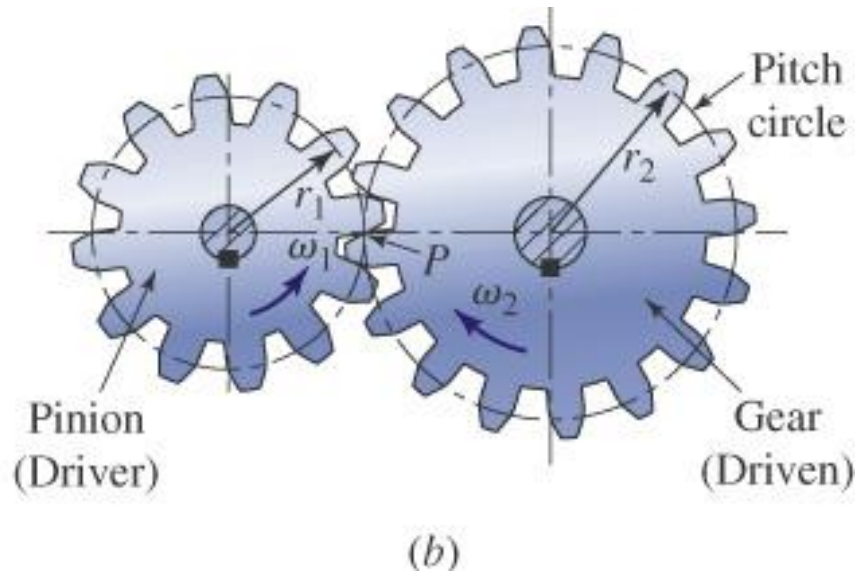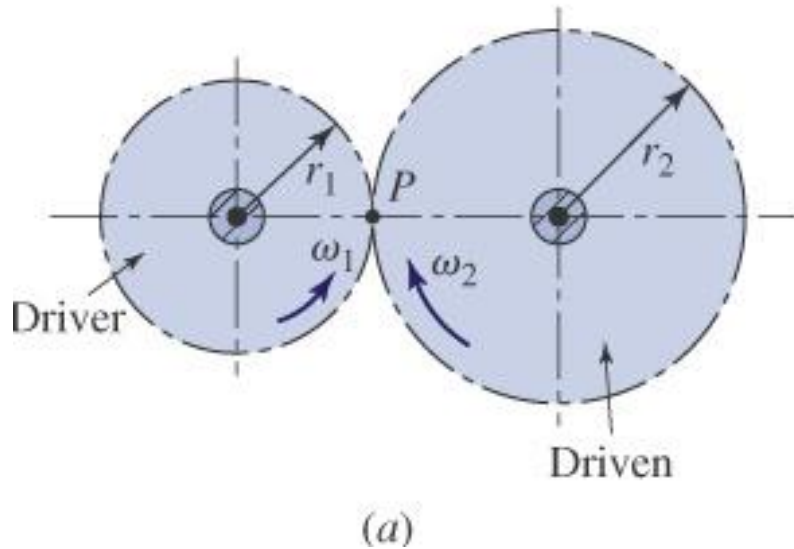


Smaller Gear is Pinion and Larger one is the gear

In most application the pinion is the driver, This reduces speed but it increases torque.

# Internal Spur Gear System



(a)

(b)

Face width, Addendum circle, Top land, Face, Addendum, Circular pitch p, Pitch circle, Flank, Tooth thickness, Width of space, Working depth, Dedendum, Clearance, Fillet radius, Dedendum circle, Clearance circle, Bottom land
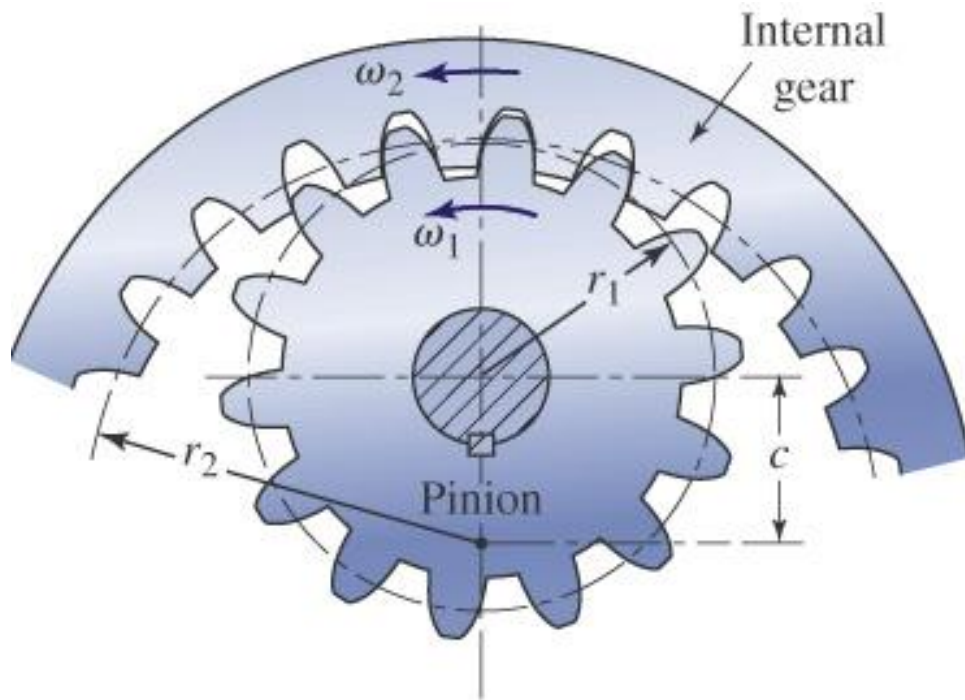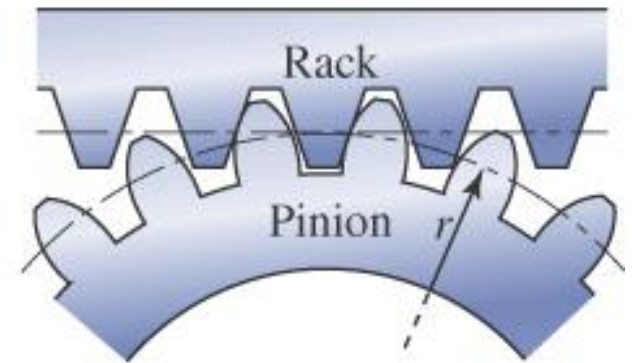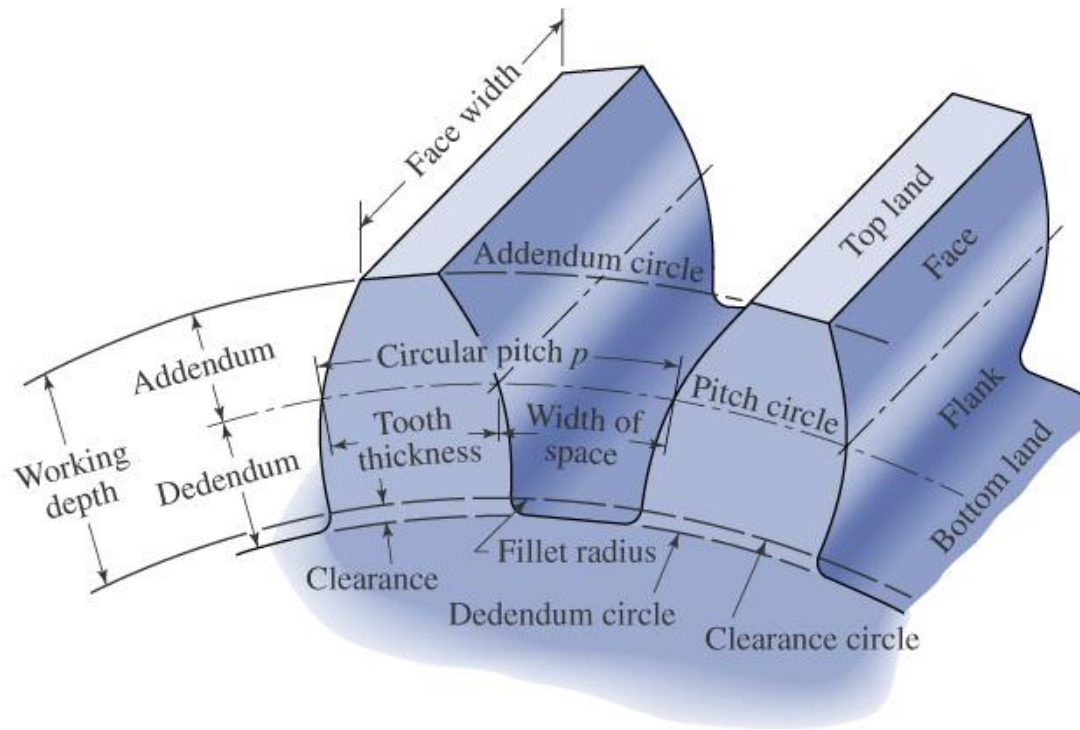
pitch circle, theoretical circle upon which all calculation is based

p, Circular pitch, p the distance from one teeth to the next, along the pitch circle. p=πd/N

m, module=d/N pitch circle/number of teeth

p= πm

P, Diametral Pitch P=N/d

pP= π
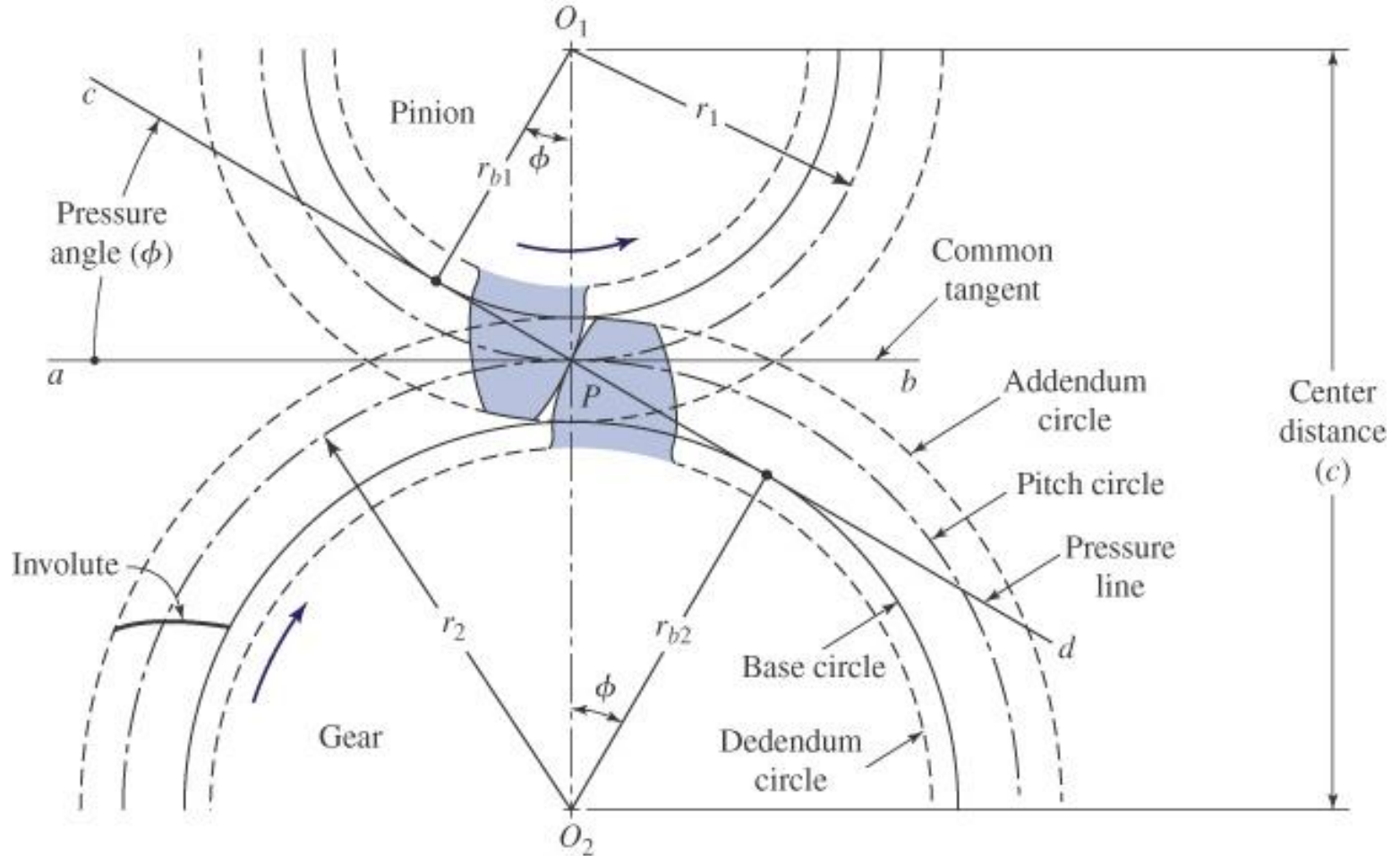
Angle Φ has the values of 20 or 25 degrees. Angle 14.5 have been also used.

Gear profile is constructed from the base circle. Then additional clearance are given.

# How Gear Profile is constructed
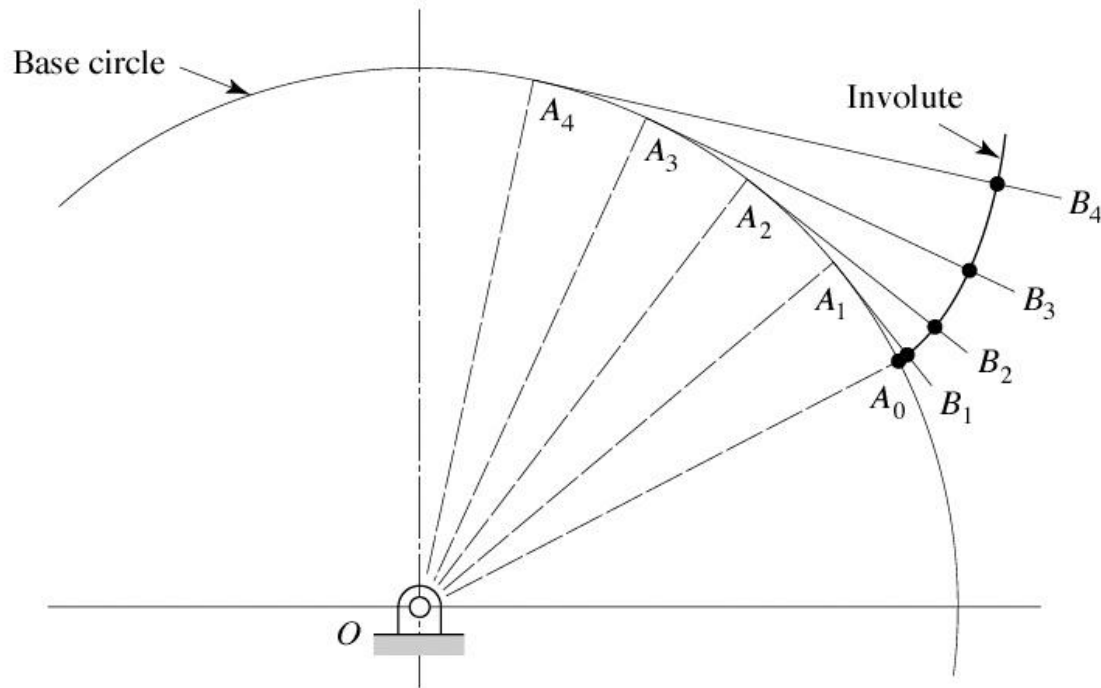
$A_1B_1 = A_1A_0$, $A_2B_2 = 2 A_1A_0$ , etc

# Standard Gear Teeth

| Item | $20^o$ full depth | $20^o$ Stub | $25^o$ full depth |
|---|---|---|---|
| Addendum a | 1/P | 0.8/P | 1/P |
| Dedendum | 1.25/P | 1/P | 1.25/P |
| Clearance f | 0.25/P | 0.2/P | 0.25/P |
| Working depth | 2/P | 1.6/P | 2/P |
| Whole depth | 2.25/P | 1.8/P | 2.25/P |
| Tooth thickness | 1.571/P | 1.571/P | 1.571/P |
| Face width | 9/P<b<13/P | 9/P<b<13/P | 9/P<b<13/P |

# Gear Trains
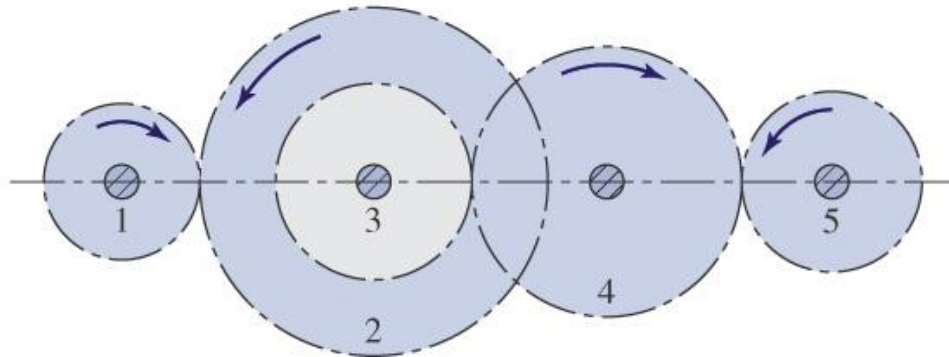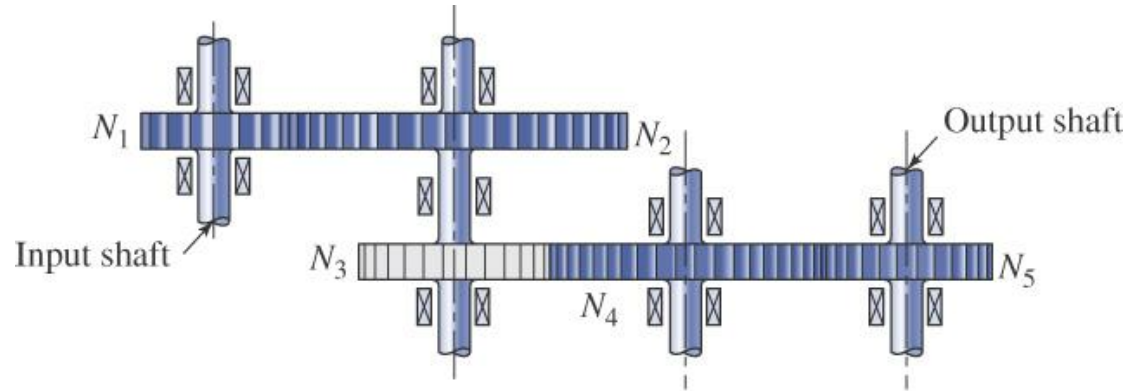


$$\frac{n_5}{n_1} = (-\frac{N_1}{N_2})(-\frac{N_3}{N_4})(-\frac{N_4}{N_5})$$

# Planetary Gear train
## You can get high torque ratio in a smaller space
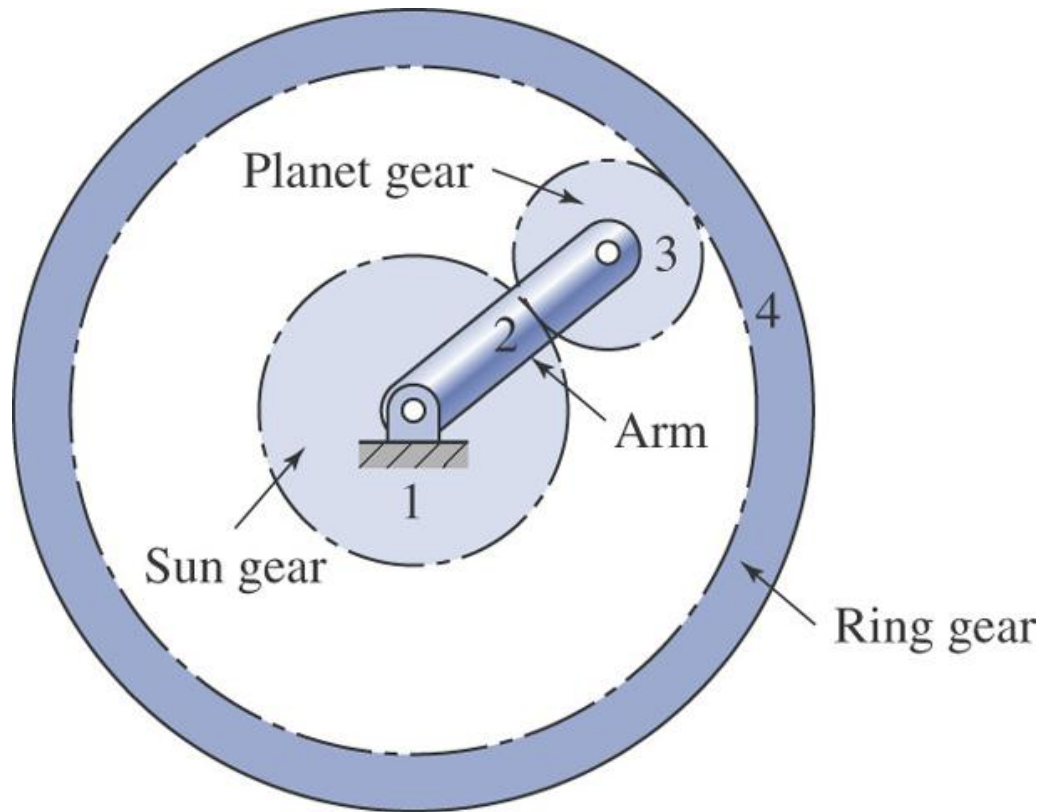


There are two inputs to the planetary gears, RPM of sun and Ring,
The out put is the speed of the arm.

# Example of planetary Gear train



Planet gear

3

4

Arm

2

1

Sun gear
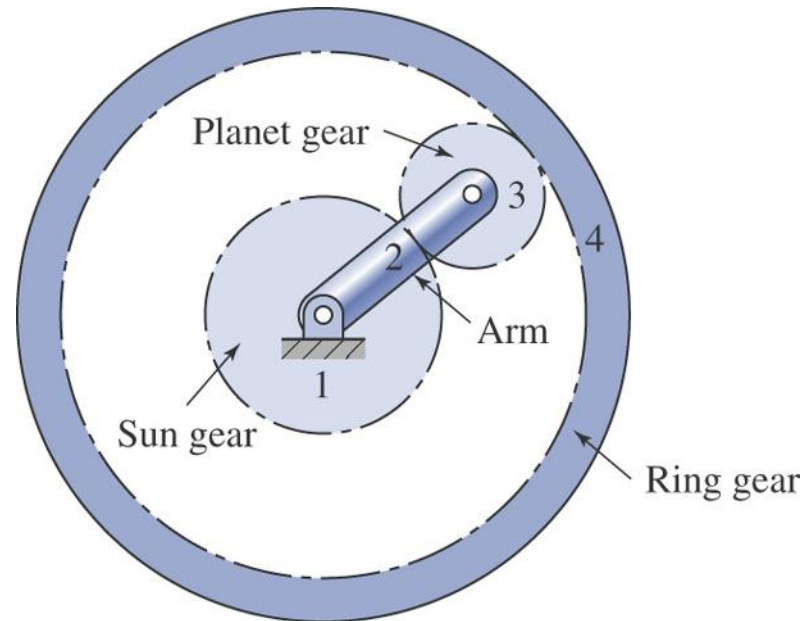
Ring gear

Gear 1, sun , RPM 1200, Number of teeth 20,

Planet Gear , Number of teeth 30

Ring Gear, Rotates RPM 120, and teeth of 80,

¼ horse power, find the speed of the arm and torque on the ring.

Alternatively you may have Certain Out put Torque requirements

# Transmitted Load

- With a pair of gears or gear sets, Power is transmitted by the force developed between contacting Teeth
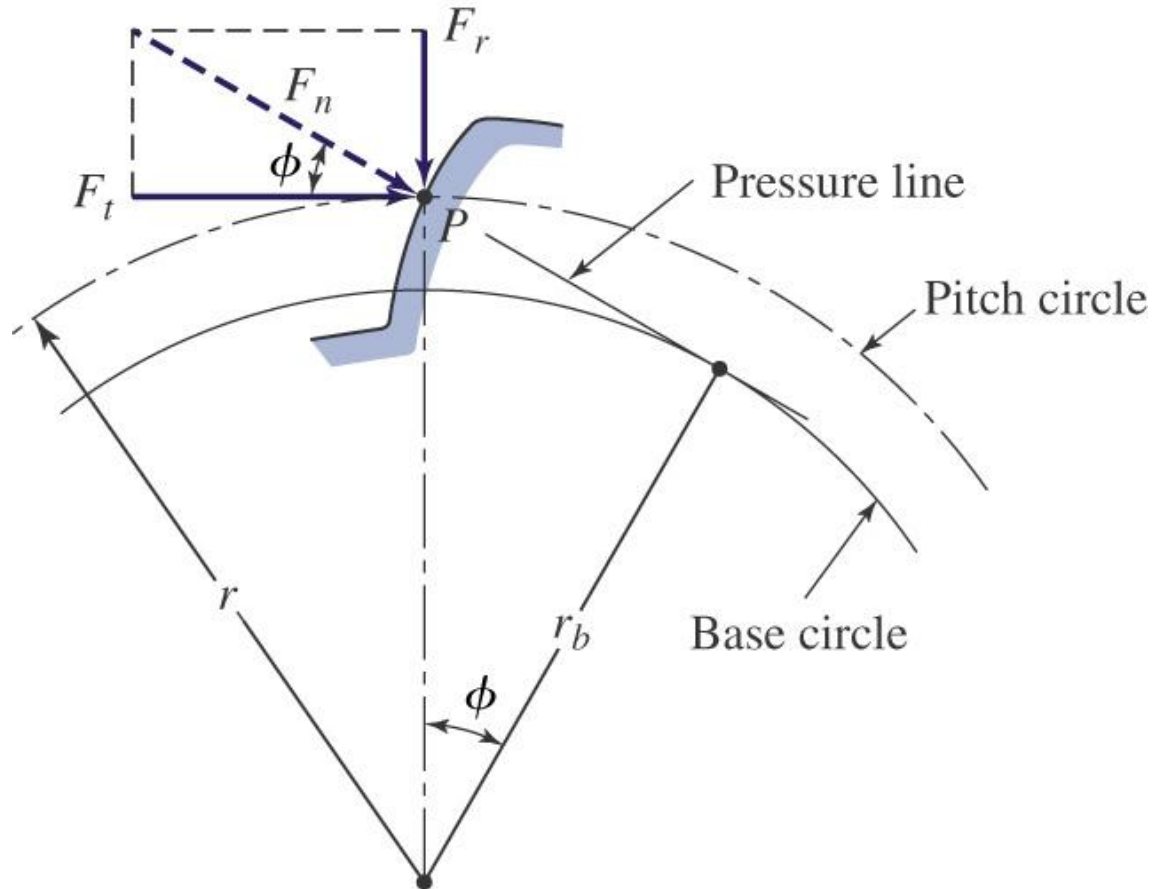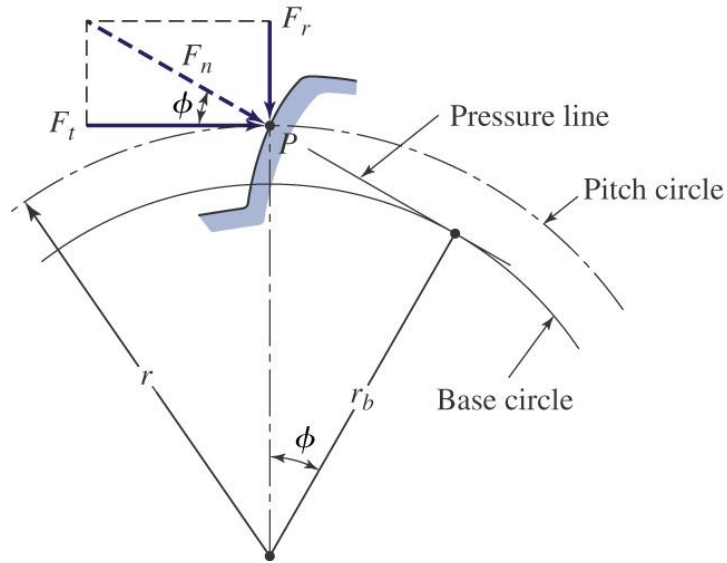
$$F_t = F_n \cos \phi$$

$$F_r = F_n \sin \phi$$

$$V = d/2\omega = d * \frac{2\pi RPM}{60} \qquad \text{d in, RPM rev./min, V in/sec}$$

$$V = \frac{\pi dn}{12} \qquad \text{d in, n rpm, V fpm}$$

$$hp = \frac{Tn}{63000} \qquad \text{Toque lb-in}$$

$$F_t = \frac{33000hp}{V} \qquad \text{V fpm}$$

$$KW = \frac{F_t V}{1000} = \frac{Tn}{9549} \qquad \text{T= N.m, V m/s, F Newton}$$

These forces have to be corrected for dynamic effects , we discuss later, considering AGMA factors

# Some Useful Relations

- F=33000hp/V   V fpm     English system

- Metric System
- KW=(FV)/1000=Tn/9549
- F newton,  V m/s,  n rpm, T,  N.m
- hp= FV/745.7=Tn/7121

# Bending Strength of the a Gear Tooth

$$\sigma = \frac{Mc}{I} = \frac{(F_t L)t/2}{bt^3/12} = \frac{6F_t}{bt^2}$$



(a)

(b)
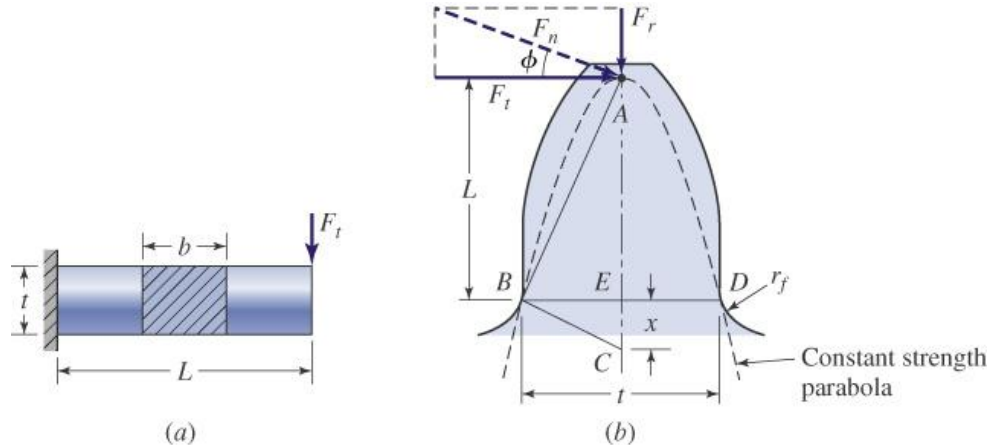
Earlier Stress Analysis of the Gear Tooth was based on

A full load is applied to the tip of a single tooth

The radial load is negligible

The load is uniform across the width

Neglect frictional forces

The stress concentration is negligible

This equation does not consider stress concentration, dynamic effects, etc.

# Design for the Bending Strength of a Gear Tooth: The AGMA Method

$$\sigma = F_t K_0 K_v \frac{P}{b} \frac{K_s K_m}{J}$$  U.S. Customary

$$\sigma = F_t K_0 K_v \frac{1.0}{bm} \frac{K_s K_m}{J}$$  SI units

$\sigma =$   Bending stress at the root of the tooth

$F_t =$   Transmitted tangential load

$K_0 =$   Overload factor

$K_v =$   Velocity factor

$P =$   Diameteral pitch, P

$b =$   Face width

$m =$   Metric modue

$K_s =$   Size factor

$K_m =$   Mounting factor

$J =$   Geometry factor

# Your stress should not exceed allowable stress

$$\sigma_{all} = \frac{S_t K_L}{K_T K_R}$$

$\sigma_{all} =$ Allowable bending stress

$S_t =$ Bending Strength

$K_L =$ Life factor

$K_T =$ Temperature factor

$K_R =$ Reliability factor

# Overload Factor - $K_o$

Table 11.4    Overload correction factor $K_o$

| Source of power | Load on driven machine | | |
|---|---|---|---|
| | Uniform | Moderate shock | Heavy shock |
| Uniform | 1.00 | 1.25 | 1.75 |
| Light shock | 1.25 | 1.50 | 2.00 |
| Medium shock | 1.50 | 1.75 | 2.25 |

# Dynamic Factor - $K_v$
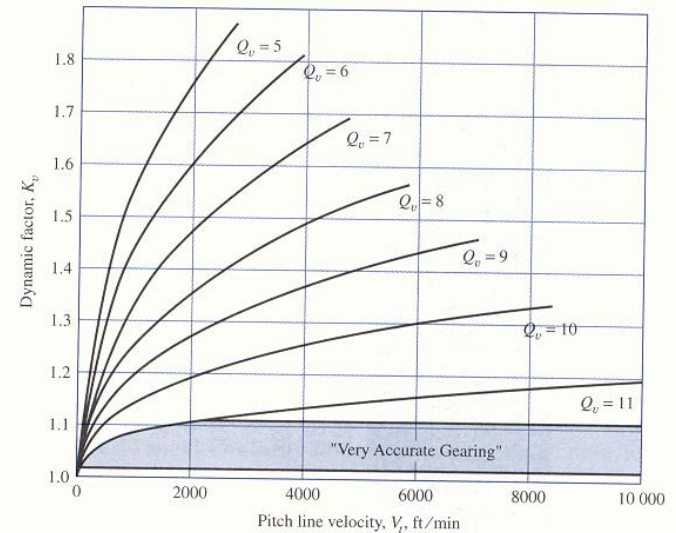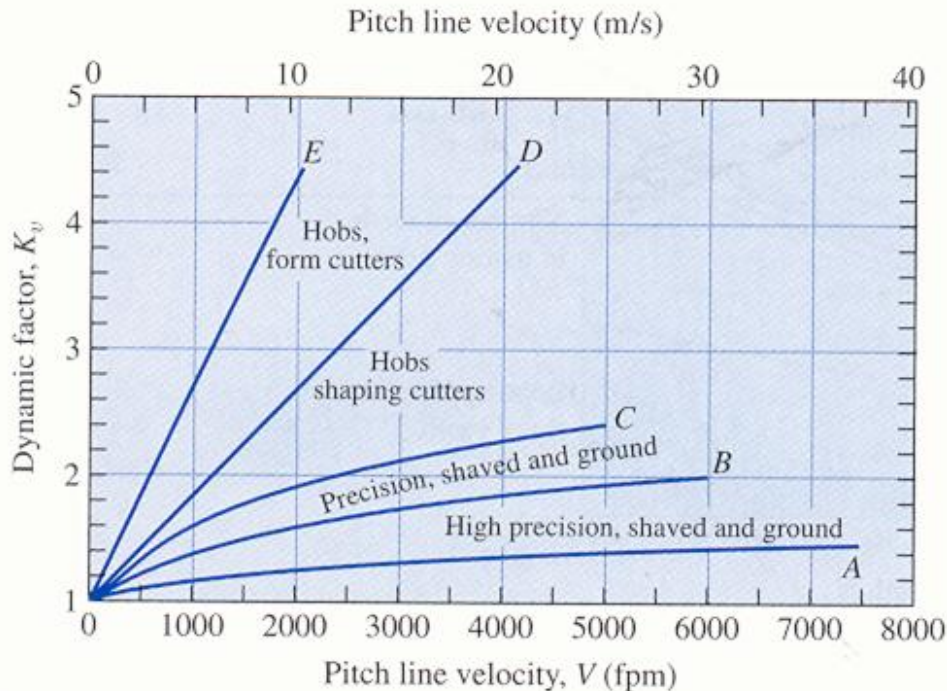


-Even with steady loads tooth impact can cause shock loading
-Impact strength depends on quality of the gear and the speed of
 gear teeth (pitch line velocity)
-Gears are classified with respect to manufacturing tolerances:
      -$Q_v$ 3 – 7, commercial quality
      -$Q_v$ 8 – 12, precision
-Graphs are available which chart $K_v$ for different quality factors

# Load Distribution Factor - $K_m$

**Table 11.5**     Mounting correction factor $K_m$

| Condition of support | Face width (in.) | | | |
| --- | --- | --- | --- | --- |
| | 0 to 2 | 6 | 9 | 16 up |
| Accurate mounting, low bearing clearances, maximum deflection, precision gears | 1.3 | 1.4 | 1.5 | 1.8 |
| Less rigid mountings, less accurate gears, contact across the full face | 1.6 | 1.7 | 1.8 | 2.2 |
| Accuracy and mounting such that less than full-face contact exists | | Over 2.2 | | |

-Failure greatly depends on how load is distributed across face
       -Accurate mounting helps ensure even distribution
-For larger face widths even distribution is difficult to attain
-Note formula depends on face width which has to be estimated for initial iteration
       -Form goal: $b < D_p$; $6 < b*P < 16$

# Reliability Factor - $K_R$

**Table 11.8**     Reliability factor $K_R$

| Reliability (%) | 90 | 99 | 99.9 | 99.99 |
|---|---|---|---|---|
| Factor $K_R$ | 0.85 | 1.00 | 1.25 | 1.50 |

I SOURCE: The AGMA.

-Adjusts for reliability other than 99%

- $K_R = 0.658 - 0.0759 \ln(1-R)$   $0.5 < R < 0.99$
- $K_R = 0.50 - 0.109 \ln(1-R)$     $0.99 < R < 0.9999$

# AGMA Geometry Factor - J
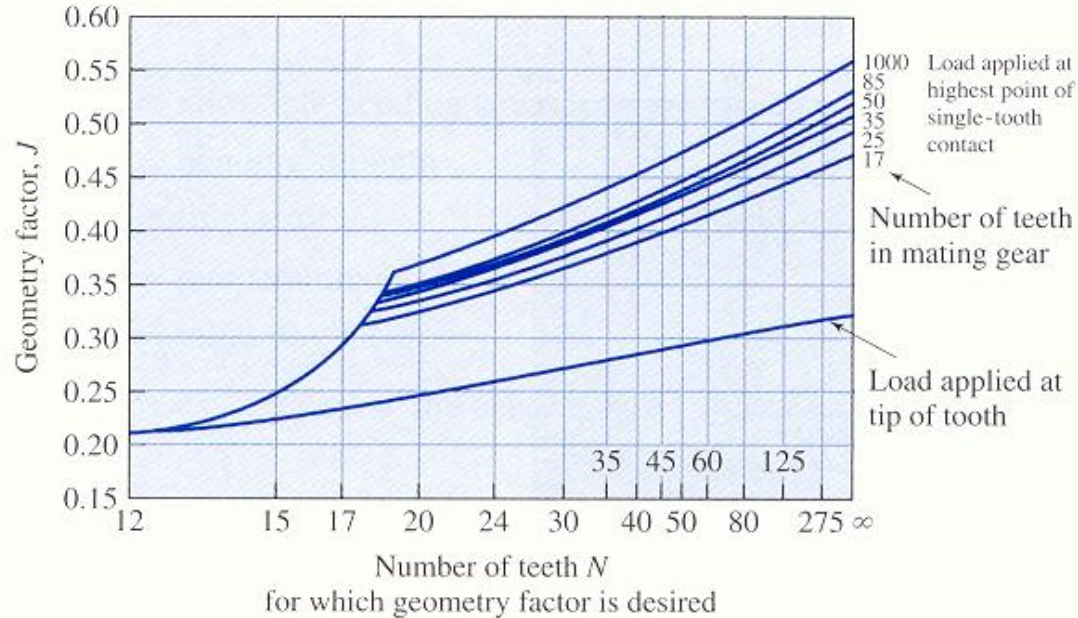


-Updated Lewis Form Factor includes effect of stress concentration at fillet
    -Different charts for different pressure angles
-Available for Precision Gears where we can assume load sharing (upper curves)
    -HPSTC – highest point of single tooth contact
    -Account for meshing gear and load sharing (contact ratio > 1)
-Single tooth contact conservative assumption (bottom curve)
    -J = 0.311 ln N + 0.15 (20 degree)
    -J = 0.367 ln N + 0.2016 (25 degree)

# Bending Strength No. – $S_t$, Fatigue bending strength

**Table 11.6**    Bending strength $S_t$ of spur, helical, and bevel gear teeth

| Material | Heat treatment | Minimum hardness or tensile strength | $S_t$ ksi | $S_t$ (MPa) |
|---|---|---|---|---|
| Steel | Normalized | 140 Bhn | 19–25 | (131–172) |
| | Q&T | 180 Bhn | 25–33 | (172–223) |
| | Q&T | 300 Bhn | 36–47 | (248–324) |
| | Q&T | 400 Bhn | 42–56 | (290–386) |
| | Case carburized | 55 $R_C$ | 55–65 | (380–448) |
| | | 60 $R_C$ | 55–70 | (379–483) |
| | Nitrided AISI-4140 | 48 $R_C$ case | 34–45 | (234–310) |
| | | 300 Bhn core | | |
| Cast iron | | | | |
|   AGMA Grade 30 | | 175 Bhn | 8.5 | (58.6) |
|   AGMA Grade 40 | | 200 Bhn | 13 | (89.6) |
| Nodular iron ASTM Grade: | | | | |
|   60-40-18 | | | 15 | (103) |
|   80-55-06 | Annealed | | 20 | (138) |
|   100-70-18 | Normalized | | 26 | (179) |
|   120-90-02 | Q&T | | 30 | (207) |
| Bronze, AGMA 2C | Sand cast | 40 ksi (276 MPa) | 5.7 | (39.3) |

SOURCE: AGMA 218.01.

Q&T = Quenched and tempered.

-Tabulated Data similar to fatigue strength
-Range given because value depends on Grade
-Based on life of $10^7$ cycles and 99% reliability

# S$_t$ – Analytical Estimate
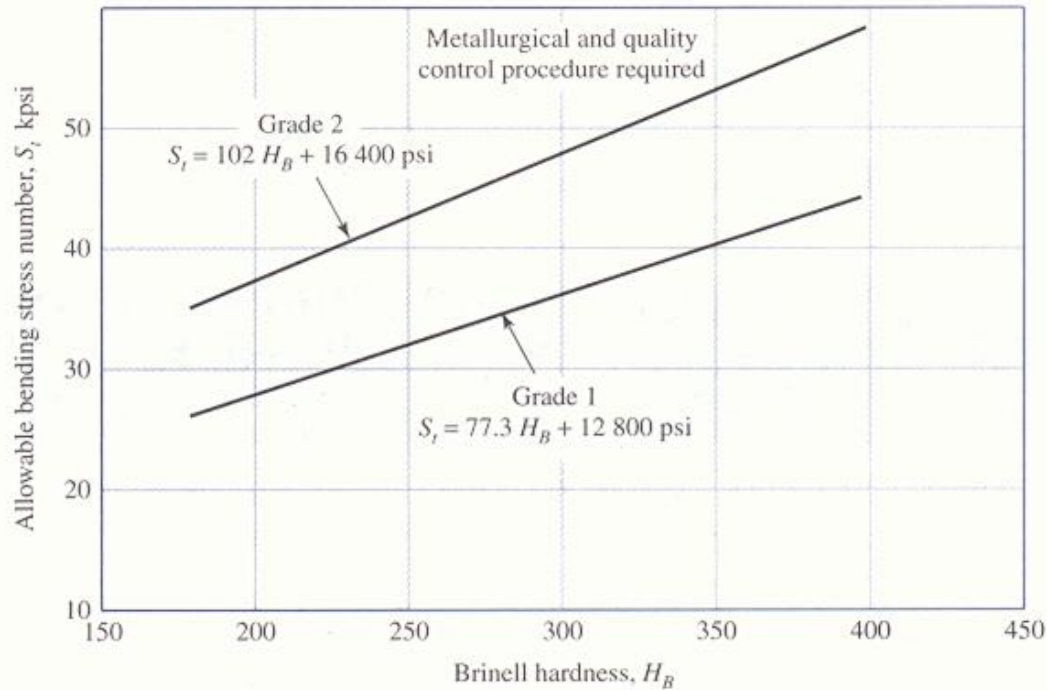


-Through hardened steel gears
       -Different charts for different manufacturing methods
-Grade 1 – good quality
     St = 77.3 H$_B$ + 12,800
-Grade 2 – premium quality
     St = 102 H$_B$ + 16,400

# Bending Strength Life Factor- $K_L$

**Table 11.7**  Life factor $K_L$ for spur and helical steel gears

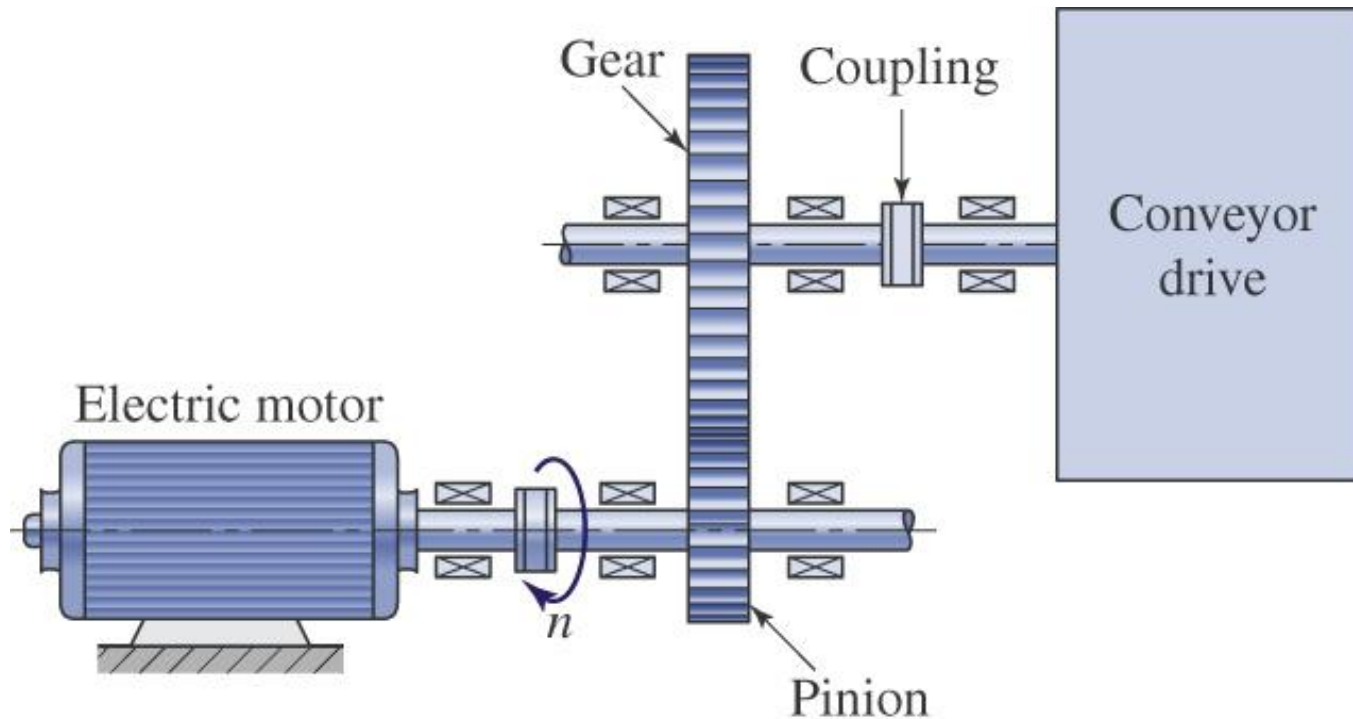| Number of cycles | 160 Bhn | 250 Bhn | 450 Bhn | Case carburized (55–63 $R_C$) |
|---|---|---|---|---|
| $10^3$ | 1.6 | 2.4 | 3.4 | 2.7–4.6 |
| $10^4$ | 1.4 | 1.9 | 2.4 | 2.0–3.1 |
| $10^5$ | 1.2 | 1.4 | 1.7 | 1.5–2.1 |
| $10^6$ | 1.1 | 1.1 | 1.2 | 1.1–1.4 |
| $10^7$ | 1.0 | 1.0 | 1.0 | 1.0 |

SOURCE: AGMA 218.01.
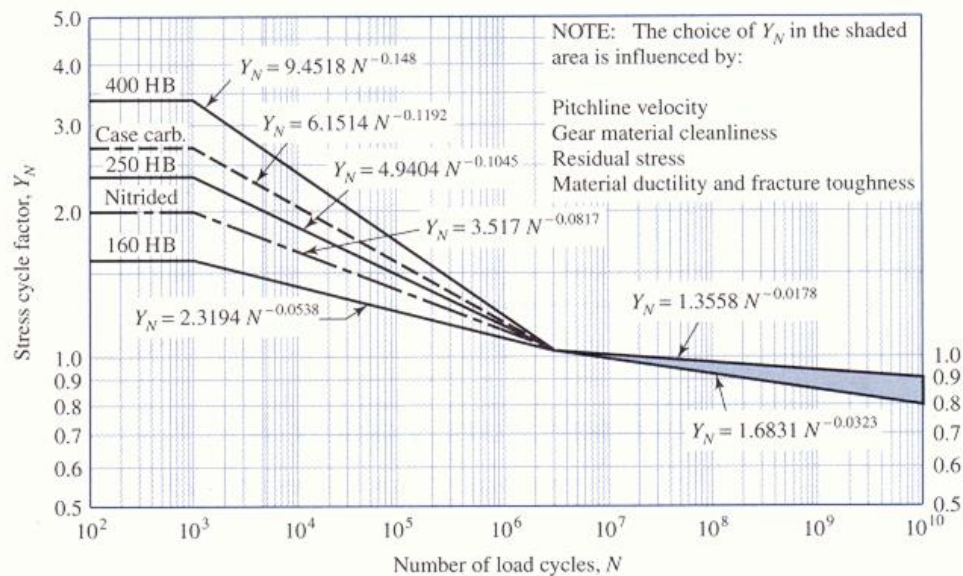


-Adjusts for life goals other than $10^7$ cycles
-Fatigue effects vary with material properties and surface finishes
-$K_L = 1.6831\ N^{-0.0323}$ N>3E6

Note: @ 2000 rpm reach 3 million cycles in 1 day of service

Example:
A conveyor drive involving heavy-shock torsional loading is operated by an electric motor, the speed ratio is 1:2 and the pinion has Diameteral pitch $P=10$ in$^{-1}$, and number of teeth $N=18$ and face width of $b=1.5$ in. The gear has Brinnel hardness of 300 Bhn. Find the maximum horspower that can be transmitted, using AGMA formula.

# Gear Box Design